Sahar Guess paper
Business Maths (1429) *for* B.A/B.Com

Q1: $(t - 3)/2 = (4 - 3t)/4$

Solution:

$$\begin{aligned}
(t-3)/2 &= (4-3t)/4 \\
4\times(t-3)/2 &= 4\times(4-3t)/4 \\
2(t-3) &= 4-3t \\
2t-6 &= 4-3t \\
2t+3t &= 4+6 \\
5t &= 10 \\
t &= \frac{10}{5} \\
t &= 2 \quad \text{Ans.}
\end{aligned}$$

Q1.1: $3(12 - x) = 16 = 2$

Solution:

$$\begin{aligned}
3(12-x)-16 &= 2 \\
36-3x-16 &= 2 \\
-3x &= 2+16-36 \\
-3x &= 18-36 \\
-3x &= -18 \\
x &= \frac{-18}{-3} \\
x &= 6 \quad \text{Ans.}
\end{aligned}$$

Q1.2: $2(y + 1) - 3(y - 1) = 5 - y$

Solution:

$$\begin{aligned}
2(y+1)-3(y-1) &= 5-y \\
2y+2-3y+3 &= 5-y \\
2y-3y+y &= 5-2-3 \\
3y-3y &= 5-5 \\
0 &= 0
\end{aligned}$$

No Solution.

Q1.3: $3x + 1 = 2 - (x - 4) + 3x$

Solution:

$$\begin{aligned} 3x + 1 &= 2 - (x - 4) + 3x \\ 3x + 1 &= 2 - x + 4 + 3x \\ 3x + 1 &= 3x - x + 6 \\ 3x + 1 &= 2x + 6 \\ 3x - 2x &= 6 - 1 \\ x &= 5 \quad \text{Ans.} \end{aligned}$$

Q1.4: $3(x - 2) + 4(2 - x) = x + 2(x + 1)$

Solution:

$$\begin{aligned} 3(x-2) + 4(2-x) &= x + 2(x + 1) \\ 3x - 6 + 8 - 4x &= x + 2x + 2 \\ -x + 2 &= 3x + 2 \\ -x - 3x &= 2-2 \\ -4x &= 0 \\ x &= \frac{0}{-4} \\ x &= 0 \quad \text{Ans.} \end{aligned}$$

Q2: $3r^2 = 14r - 8$

Solution:

$3r^2 = 14r - 8$

$3r^2 - 14r + 8 = 0$

Compare it with

$ar^2 + br + c = 0$

$a = 3, b = -14, c = 8$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-14) \pm \sqrt{(-14)^2 - 4(3)(8)}}{2(3)}$$

$$= \frac{14 \pm \sqrt{196 - 96}}{6}$$

$$= \frac{14 \pm \sqrt{100}}{6}$$

$$= \frac{14 \pm 10}{6}$$

$$x = \frac{14-10}{6} \quad \text{or} \quad x = \frac{14+10}{6}$$

$$x = \frac{4}{6} \qquad x = \frac{24}{6}$$

$$x = \frac{2}{3} \qquad x = 4 \text{ Ans.}$$

Q2.1: $x^2 = 2x - 2$

Solution:

$x^2 = 2x - 2$

$x^2 - 2x + 2 = 0$

Compare it with

$ax^2 + bx + c = 0$

$a = 1, b = -2, c = 2$

We know that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4-8}}{2}$$

$$= \frac{2 \pm \sqrt{-6}}{2}$$

Solution is not possible.

Q2.2: $4t^2 + 3t = 1$

Solution:

$4t^2 + 3t = 1$

$4t^2 + 3t - 1 = 0$

Compare it with

$at^2 + bt + c = 0$

$a = 4, b = 3, c = -1$

We know that

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{(3)^2 - 4(4)(-1)}}{2(4)}$$

$$= \frac{-3 \pm \sqrt{9+16}}{8}$$

$$= \frac{-3 \pm \sqrt{25}}{8}$$

$$= \frac{-3 \pm 5}{8}$$

$$t = \frac{-3-5}{8} \quad \text{or} \quad t = \frac{-3+5}{8}$$

$$t = \frac{-8}{8} \qquad t = \frac{2}{8}$$

$$t = -1 \qquad t = \frac{1}{4}$$

Q2.3: $y^2 + 2 = 2y$

Solution:

$y^2 + 2 = 2y$

$y^2 - 2y + 2 = 0$

Compare it with

$ay^2 + by + c = 0$

$a = 1, b = -2, c = 2$

We know that

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4-8}}{2}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

Solution is not possible.

Q3: $2x^2 - 3x - 2 < 0$

Solution:

$2x^2 - 3x - 2 < 0$

$2x^2 - 4x + x - 2 < 0$ As $2x(-2) = -4$

$2x(x-2)+1(x-2) < 0$ So $-3x = -4x + x$

$(2x+1)(x-2) < 0$

The following attributes of the two factors on the left side will result in the inequality will be negative if the two factors have the opposite sign.

Factors			
Product	(x - 2)	(2x + 1)	Condtions
< 0	< 0	> 0	Condition - 1
< 0	> 0	< 0	Condition - 2

Condition - 1:

$2x + 1 > 0$ and $x - 2 < 0$

$2x > -1$ $x < 2$

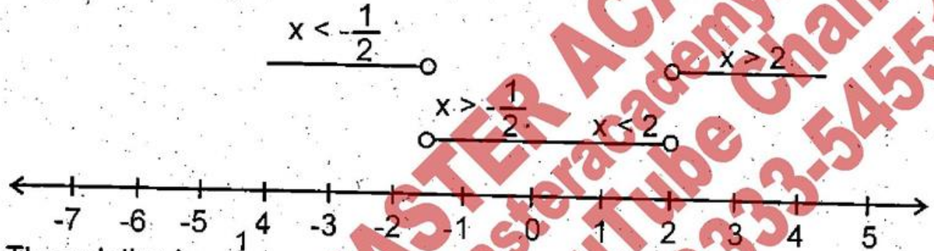
$x > -\frac{1}{2}$

Condition - 2:

$2x + 1 < 0$ and $x - 2 > 0$

$2x < -1$ $x > 2$

$x < -\frac{1}{2}$



The solution is $-\frac{1}{2} < x < 2$.

Q3.1: $2x^2 - x - 10 > 0$

Solution:

$2x^2 - x - 10 > 0$

$2x^2 - 5x + 4x - 10 > 0$ As $2x(-10) = -20$

$x(2x-5)+2(2x-5) > 0$ So $-x = -5x + 4x$

$(x+2)(2x-5) > 0$

The following attributes of the two factors on the left side of the inequality will be positive if the two factors have the same sign.

	Factors		
Product	(2x - 5)	(x + 2)	Condtions
> 0	> 0	> 0	Condition - 1
> 0	< 0	< 0	Condition - 2

Condition - 1:

$x + 2 > 0$ and $2x - 5 > 0$

$x > -2$ $2x > 5$

$x > \frac{5}{2}$

Condition - 2:

$x + 2 < 0$ and $2x - 5 < 0$

$x < -2$ $2x < 5$

$x < \frac{5}{2}$

The solution is $x < -2$ or $x > \frac{5}{2}$

Q3.2: $-3x + 4y - 10 = 7x - 2y + 50$

Solution:

$-3x + 4y - 10 = 7x - 2y + 50$

$-3x - 7x + 4y - 2y = 50 + 10$

$-10x + 2y = 60$ ---------- (1)

For x - intercept:

Put y = 0 in eq.(1), we get

$-10x + 2(0) = 60$

$-10x = 60$

$x = \frac{60}{-10}$

$x = -6$ (-6,0)

For y-intercept:

Put x = 0 in eq.(1), we get

$-10(0) + 2y = 60$

$2y = 60$

$y = \frac{60}{2}$

$y = 30$ (0,30)

Q3.3: $15y - 90 = 0$

Solution:

$15y - 90 = 0$ ------------ (1)

For x - intercept:

Put y = 0 in eq.(1), we get

$15(0) - 90 = 0$

$-90 = 0$

No solution.

For y-intercept:

Put x = 0 in eq.(1), we get

$15y - 90 = 0$

$15y = 90$

$y = \frac{90}{15}$

$y = 6$ (0,6)

Q3.4: $(x-2y)/3 - 12 = (2x + 4y)/3$

Solution:

$(x - 2y/3 - 12 = (2x + 4y)/3$

$x - 2y - 36 = 2x + 4y$

$x - 2x - 2y - 4y = 36$

$-x - 6y = 36$ ---------- (1)

For x - intercept:

Put y = 0 in eq.(1), we get

$-x - 6(0) = 36$

$-x = 36$

$x = -36 \quad (-36,0)$

For y-intercept:

Put x = 0 in eq.(1), we get

$-(0) - 6y = 36$

$-6y = 36$

$y = \frac{36}{-6}$

$y = -6 \quad (0,-6)$

Q3.5: ax + by = t

Solution:

$ax + by = t$ ------------- (1)

For x - intercept:

Put y = 0 in eq.(1), we get

$ax + b(0) = t$

$ax = t$

$x = \frac{t}{a} \quad \left(\frac{t}{a},0\right)$

For y-intercept:

Put x = 0 in eq.(1), we get

$a(0) + by = t$

$by = t$

$y = \frac{t}{b} \quad \left(0,\frac{t}{b}\right)$

Q3.6: Cx - dy = e

Solution:

$Cx - dy = e$ ------------- (1)

For x - intercept:

Put y = 0 in eq.(1), we get

$Cx - d(0) = e$

$Cx = e$

$x = \frac{e}{c} \quad \left(\frac{e}{c},0\right)$

For y-intercept:

Put x = 0 in eq.(1), we get

$C(0) - dy = e$

$dy = e$

$y = -\frac{e}{d}$ $\left(0, -\frac{e}{d}\right)$

Q3.7: px = q

Solution:

$px = q$ --------- (1)

For x - intercept:

Put y = 0 in eq.(1), we get

$px = q$

$x = \frac{q}{p}$ $\left(\frac{q}{p}, 0\right)$

For y-intercept:

Put x = 0 in eq.(1), we get

$p(0) = q$

$0 = q$

No solution.

Q3.8: dx - ey + f = gx - hy

Solution:

$dx - ey + f = gx - hy$ --------- (1)

For x - intercept:

Put y = 0 in eq.(1), we get

$dx - e(0) + f = gx - h(0)$

$dx + f - gx$

$dx - gx = -f$

$(d-g)x = -f$

$-(g-d)x = -f$

$x = \frac{f}{(g-d)}$ $\left(\frac{f}{(g-d)}, 0\right)$

For y-intercept:

Put x = 0 in eq.(1), we get

$d(0) - ey + f = g(0) - hy$

$-ey + f = -hy$

$-ey + hy = -f$

$-(ey - hy) = -f$

$-(e-h)y = -f$

$y = \frac{f}{e-h}$, $\left(0, \frac{f}{e-h}\right)$

Q3.9: -ry = s

Solution:

$-ry = s$ --------- (1)

For x - intercept:

Put y = 0 in eq.(1), we get

$-r(0) = s$

$0 = s$

No solution:

For y-intercept:

Put x = 0 in eq.(1), we get

$-ry = s$

$y = -\frac{s}{r}$ $\left(0, -\frac{s}{r}\right)$

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Q4: Slope = $\frac{3}{2}$, (-5,-8) lies on the line.

Solution:

Here $m = \frac{3}{2}$, $(x1, y1) = (-5, -8)$

The point slope formula is

$y - y1 = m(x - x1)$

$y - (-8) = \frac{3}{2}[x - (-5)]$

$y + 8 = \frac{3}{2}(x + 5)$

$2(y+8) = 3(x+5)$

$2y + 16 = 3x + 15$

$2y = 3x + 15 - 16$

$2y = 3x - 1$

$y = \frac{3}{2}x - \frac{1}{2}$

Q4.1: Slope = 0, (20, -10) lies on line.

Solution:

Here m = 0, (x1, y1) = (20, -10)

The point slope formula is

$y - y_1 = m(x - x_1)$

$y - (-10) = 0(x - 20)$

$y + 10 = 0$

$y = -10$

Q4.2: Passes through (7, 2) and is parallel to the line (a) x = 7 and (b) y = 6

Solution; (a)

Here $(x_1, y_1) = (7, 2)$

And $x = 7$

$0 = -x + 7$

It cannot be compared with

$y = mx + k$

$m = \infty$ (Undefined)

The slope - intercept formula is

$y - y_1 = m(x - x_1)$

$y - 2 = \infty(x - 7)$

$y - 2 = \frac{1}{0}(x - 7)$

$0(y-2) = 1(x - 7)$

$0 = x - 7$

or $x = 7$

Solution: (b)

Here $(x_1, y_1) = (7, 2)$

And $y = 6$

$y = 0x + 6$

Compare it with

$y = mx + k$

$m = 0$

The slope - intercept formula is

$y - y1 = m(x - x1)$

$y - 2 = 0(x - 7)$

$y - 2 = 0$

$y = 2$

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Q5: Rewrite the following systems of equations in matrix form.

ax1 + bx2 = c

dx1 + ex2 = f

gx1 + hx2 = i

Solution:

ax1 + bx2 = c

dx1 + ex2 = f

gx1 + hx2 = i

In Matrix form:

$$\begin{bmatrix} a & b \\ d & e \\ g & h \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} c \\ f \\ i \end{bmatrix}$$

Q5.1: Rewrite the following systems of equations in matrix form.

ax1 + bx2 + cx3 + dx4 + ex5 = i

gx1- hx3 + ix5 = j

Solution:

ax1 + bx2 + cx3 + dx4 + ex5 = i

gx1 - hx3 + ix5 = j

In Matrix form:-

$$\begin{bmatrix} a & b & c & d & e \\ g & o & -h & o & i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} i \\ j \end{bmatrix}$$

Q5.2: Rewrite the following systems of equations in matrix form.

$a1x^2 + a2x + a3 = b1$

$a4x^2 + a5x + a6 = b2$

Solution:

$$a1x^2 + a2x + a3 = b1$$
$$a4x^2 + a5x + a6 = b2$$

In Matrix form:

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix} \begin{bmatrix} x^2 \\ x \\ 1 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Q5.3: $\begin{bmatrix} 2 & -4 & -2 \\ -2 & 0 & 4 \\ 4 & 3 & -3 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 2 & -4 & -2 \\ -2 & 0 & 4 \\ 4 & 3 & -3 \end{bmatrix}$

The Cofactors are

$a'11 = (-1)1+1 \begin{vmatrix} 0 & 4 \\ 3 & -3 \end{vmatrix} = +(0-12) = -12$

$a'12 = (-1)1+2 \begin{vmatrix} -2 & 4 \\ 4 & -3 \end{vmatrix} = -(6-16) = -(-10) = 10$

$a'13 = (-1)1+3 \begin{vmatrix} -2 & 0 \\ 4 & -3 \end{vmatrix} = +(-6-0) = -6$

$a'21 = (-1)2+1 \begin{vmatrix} -4 & -2 \\ 3 & -3 \end{vmatrix} = -(12+6) = -18$

$a'22 = (-1)2+2 \begin{vmatrix} 2 & -2 \\ 4 & -3 \end{vmatrix} = +(-6+8) = 2$

$a'23 = (-1)3+2 \begin{vmatrix} 2 & -4 \\ 4 & -3 \end{vmatrix} = -(6+16) = -22$

$a'31 = (-1)3+2 \begin{vmatrix} -4 & -2 \\ 0 & 4 \end{vmatrix} = +(-16-0) = -16$

$a'32 = (-1)3+2 \begin{vmatrix} 2 & -2 \\ -2 & 4 \end{vmatrix} = -(8-4) = -4$

$a'33 = (-1)3+3 \begin{vmatrix} 2 & -4 \\ -2 & 0 \end{vmatrix} = +(0-8) = -8$

The cofactor matrix is

$$Ac = \begin{bmatrix} -12 & 10 & -6 \\ -18 & 2 & -22 \\ -16 & -4 & -8 \end{bmatrix}$$

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Q6: Find the determinent of

$$A = \begin{bmatrix} 2 & 7 & -2 & 0 \\ 1 & -2 & -3 & 0 \\ 3 & 3 & 6 & 9 \\ 6 & -3 & -2 & 0 \end{bmatrix}$$

Solution:

$$|A| = \begin{bmatrix} 2 & 7 & -2 & 0 \\ 1 & -2 & -3 & 0 \\ 3 & 3 & 6 & 9 \\ 6 & -3 & -2 & 0 \end{bmatrix}$$

Expand from Column 4.

$$= 0 - 0 + 9\begin{vmatrix} 2 & 7 & -2 \\ 1 & -2 & -3 \\ 6 & -3 & -2 \end{vmatrix} - 0$$

$$= 9\left[2\begin{vmatrix} -2 & -3 \\ -3 & -2 \end{vmatrix} - 7\begin{vmatrix} 1 & -3 \\ 6 & -2 \end{vmatrix} + (-2)\begin{vmatrix} 1 & -2 \\ 6 & -3 \end{vmatrix}\right]$$

$$= 9\left[2(4-9) - 7(-2+18) - 2(-3+12)\right]$$

$$= 9\left[2(-5) - 7(16) - 2(9)\right]$$

$$= 9\left[-10 - 112 - 18\right]$$

$$= 9\left[-140\right]$$

$$= -1260$$

In the following exercises, solve the system of equations by using Cramer's rule.

Q6.1: 3x1 - 2x2 = -13

4x1 + 6x2 = 0

Solution:

3x1 - 2x2 = -13

4x1 + 6x2 = 0

In Matrix form:

$$\begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -13 \\ 0 \end{bmatrix}$$

According to Cramer's rule:-

$$x_i = \frac{\Delta_i}{\Delta}$$

So

$$x1 = \frac{\begin{vmatrix} -13 & -2 \\ 0 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 4 & 6 \end{vmatrix}}$$

$$= \frac{(-13)\times 6-(-2)\times 0}{3\times 6-(-2)\times 4}$$

$$= \frac{-78+0}{18+8} = -\frac{78}{26} = -3$$

And

$$x2 = \frac{\begin{vmatrix} 3 & -13 \\ 4 & 0 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 4 & 6 \end{vmatrix}}$$

$$= \frac{3\times 0-(-13)\times 4}{3\times 6-(-2)\times 4}$$

$$= \frac{0+52}{18+8} = \frac{52}{26} = 2$$

Q6.2: 5x1 - 4x2 = -8

3x1 + 5x2 = 47

Solution:

5x1 - 4x2 = -8

3x1 + 5x2 = 47

In Matrix form:

$$\begin{bmatrix} 5 & -4 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -8 \\ 47 \end{bmatrix}$$

According to Cramer's rule:-

$$x1 = \frac{\begin{vmatrix} -8 & -4 \\ 47 & 5 \end{vmatrix}}{\begin{vmatrix} 5 & -4 \\ 3 & 5 \end{vmatrix}}$$

$$= \frac{-8 \times 5 - (-4) \times 47}{5 \times 5 - (-4) \times 3}$$

$$= \frac{-40 + 188}{25 + 12} = \frac{148}{37} = 4$$

$$x2 = \frac{\begin{vmatrix} 5 & -8 \\ 3 & 47 \end{vmatrix}}{\begin{vmatrix} 5 & -4 \\ 3 & 5 \end{vmatrix}}$$

$$= \frac{5 \times 47 - (-8) \times 3}{5 \times 5 - (-4) \times 3}$$

$$= \frac{235 + 24}{25 + 12} = \frac{259}{37} = 7$$

Q6.3 **x1 + 3x2 - 2x3 = 17**
2x1 - 4x2 + x3 = -16
5x1 + 2x2 - 4x3 = 21

Solution:

x1 + 3x2 - 2x3 = 17
2x1 - 4x2 + x3 = -16
5x1 + 2x2 - 4x3 = 21

In Matrix form:

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -4 & 1 \\ 5 & 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ -16 \\ 21 \end{bmatrix}$$

According to Cramer's Rule:-

$$x1 = \frac{\begin{vmatrix} 17 & 3 & -2 \\ -16 & -4 & 1 \\ 21 & 2 & -4 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & -2 \\ 2 & -4 & 1 \\ 5 & 2 & -4 \end{vmatrix}}$$

$$= \frac{17\begin{vmatrix}-4 & 1\\ 2 & -4\end{vmatrix} - 3\begin{vmatrix}-16 & 1\\ 21 & -4\end{vmatrix} + (-2)\begin{vmatrix}-16 & -4\\ 21 & 2\end{vmatrix}}{1\begin{vmatrix}-4 & 1\\ 2 & -4\end{vmatrix} - 3\begin{vmatrix}2 & 1\\ 5 & -4\end{vmatrix} + (-2)\begin{vmatrix}2 & -4\\ 5 & 2\end{vmatrix}}$$

$$= \frac{17(16-2) - 3(64-21) - 2(-32+84)}{1(16-2) - 3(-8-5) - 2(4+20)}$$

$$= \frac{17(14) - 3(43) - 2(52)}{1(14) - 3(-13) - 2(24)}$$

$$= \frac{238 - 129 - 104}{14 + 39 - 48} = \frac{238 - 233}{53 - 48}$$

$$= \frac{5}{5} = 1$$

$$x2 = \frac{\begin{vmatrix}1 & 17 & -2\\ 2 & -16 & 1\\ 5 & 21 & -4\end{vmatrix}}{\begin{vmatrix}1 & 3 & -2\\ 2 & -4 & 1\\ 5 & 2 & -4\end{vmatrix}}$$

$$= \frac{1\begin{vmatrix}-16 & 1\\ 21 & -4\end{vmatrix} - 17\begin{vmatrix}2 & 1\\ 5 & -4\end{vmatrix} + (-2)\begin{vmatrix}2 & -16\\ 5 & 21\end{vmatrix}}{1\begin{vmatrix}-4 & 1\\ 2 & -4\end{vmatrix} - 3\begin{vmatrix}2 & 1\\ 5 & -4\end{vmatrix} + (-2)\begin{vmatrix}2 & -4\\ 5 & 2\end{vmatrix}}$$

$$= \frac{1(64-21) - 17(-8-5) - 2(42+80)}{1(16-2) - 3(-8-5) - 2(4+20)}$$

$$= \frac{1(43) - 17(-13) - 2(122)}{1(14) - 3(-13) - 2(24)}$$

$$= \frac{43 + 221 - 224}{14 + 39 - 48} = \frac{264 - 244}{53 - 48}$$

$$= \frac{20}{5} = 4$$

$$x3 = \frac{\begin{vmatrix}1 & 3 & +27\\ 2 & -4 & -16\\ 5 & 2 & 21\end{vmatrix}}{\begin{vmatrix}1 & 3 & -2\\ 2 & -4 & 1\\ 5 & 2 & -4\end{vmatrix}}$$

$$= \frac{1\begin{vmatrix}-4 & -16\\ 2 & 21\end{vmatrix} - 3\begin{vmatrix}2 & -16\\ 5 & 21\end{vmatrix} + 17\begin{vmatrix}2 & -4\\ 5 & 2\end{vmatrix}}{1\begin{vmatrix}-4 & 1\\ 2 & -4\end{vmatrix} - 3\begin{vmatrix}2 & 1\\ 5 & -4\end{vmatrix} - 2\begin{vmatrix}2 & -4\\ 5 & 2\end{vmatrix}}$$

$$= \frac{1(-84+32) - 3(42+80) + 17(4+20)}{1(16-2) - 3(-8-5) - 2(4+20)}$$

$$= \frac{1(-52) - 3(122) + 17(24)}{1(14) - 3(-13) - 2(24)}$$

$$= \frac{-52-366+408}{14+39-48} = \frac{-418+408}{53-48}$$

$$= -\frac{10}{5} = -2$$

x1 = 1, x2 = 4, x3 = -2

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Q7: $\begin{bmatrix}3 & 5 & 2\\ 4 & 1 & 0\\ -9 & -15 & -6\end{bmatrix}$

Solution:

Let $A = \begin{bmatrix}3 & 5 & 2\\ 4 & 1 & 0\\ -9 & -15 & -6\end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{Adj } A$

$$|A| = \begin{vmatrix}3 & 5 & 2\\ 4 & 1 & 0\\ -9 & -15 & -6\end{vmatrix}$$

$$= 3\begin{vmatrix}1 & 0\\ -15 & -6\end{vmatrix} - 5\begin{vmatrix}4 & 0\\ -9 & -6\end{vmatrix} + 2\begin{vmatrix}4 & 1\\ -9 & -15\end{vmatrix}$$

$$= 3(-6-0) - 5(-24-0) + 2(-60+9)$$

$$= 3(-6) - 5(-24) + 2(-51)$$

$$= -18 + 120 - 102$$

$$= 120 - 120$$

$$= 0$$

As $|A| = 0$, no inverse exists.

Q7.1: $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 0 & -4 \\ 1 & 2 & 5 \end{bmatrix}$

Solution:

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 0 & -4 \\ 1 & 2 & 5 \end{bmatrix}$

$A\text{-}1 = \frac{1}{|A|} \text{ Adj } A$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 0 & -4 \\ 1 & 2 & 5 \end{vmatrix}$$

$$= 1\begin{vmatrix} 0 & -4 \\ 2 & 5 \end{vmatrix} - 1\begin{vmatrix} 3 & -4 \\ 1 & 5 \end{vmatrix} + 1\begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix}$$

$$= 1(0+8) - 1(15+4) + 1(6-0)$$

$$= 1(8) - 1(19) + 1(6)$$

$$= 8 - 19 + 6$$

$$= 14 - 15$$

$$= -5$$

The Cofactors are

$a'11 = (-1)1+1 \begin{vmatrix} 0 & -4 \\ 2 & 5 \end{vmatrix} = +(0 + 8) = 8$

$a'12 = (-1)1+2 \begin{vmatrix} 3 & -4 \\ 1 & 5 \end{vmatrix} = -(15 + 4) = -19$

$a'13 = (-1)1+3 \begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix} = +(6 - 0) = 6$

$a'21 = (-1)2+1 \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} = -(5 - 2) = -3$

$a'22 = (-1)2+2 \begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix} = +(5 - 1) = 4$

$a'23 = (-1)2+3 \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = -(2 - 1) = -1$

$a'31 = (-1)3+1 \begin{vmatrix} 1 & 1 \\ 0 & -4 \end{vmatrix} = +(-4 - 0) = -4$

a'32 $=(-1)3+2 \begin{vmatrix} 1 & 1 \\ 3 & -4 \end{vmatrix}$ $= -(4 - 3)$ $= 7$

a'33 $=(-1)3+3 \begin{vmatrix} 1 & 1 \\ 3 & 0 \end{vmatrix}$ $= +(0 - 3)$ $= -3$

The cofactor matrix is

$$Ac = \begin{bmatrix} 8 & -19 & 6 \\ -3 & 4 & -1 \\ -4 & 7 & -3 \end{bmatrix}$$

The adjoint matrix is the transpose of Ac, or

$$\text{Adj } A = \begin{bmatrix} 8 & -3 & -4 \\ -19 & 4 & 7 \\ 6 & -1 & -3 \end{bmatrix}$$

$$A\text{-}1 = \frac{1}{|A|} \text{ Adj } A$$

$$= \frac{1}{-5}\begin{bmatrix} 8 & -3 & -4 \\ -19 & 4 & 7 \\ 6 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{8}{-5} & \frac{-3}{-5} & \frac{-4}{-5} \\ \frac{-19}{-5} & \frac{4}{-5} & \frac{7}{-5} \\ \frac{6}{-5} & \frac{-1}{-5} & \frac{-3}{-5} \end{bmatrix}$$

$$= \begin{bmatrix} -1.6 & 0.6 & 0.8 \\ 3.8 & -0.8 & -1.4 \\ -1.2 & 0.2 & 0.6 \end{bmatrix}$$

================

Q8: 30, 36, 28, 18, 42, 10, 20, 52

Solution:

10, 18, 20, 28, 30, 36, 42, 52

(a) Mean $= \frac{10 + 18 + 20 + 28 + 30 + 36 + 42 + 52}{8} = \frac{236}{8} = 29.5$

(b) Median $= \left(\frac{n+1}{2}\right)^{th}$ item $= \left(\frac{8+1}{2}\right)^{th}$ item $= 4.5^{th}$ item

So, Median = $\frac{28+30}{2} = \frac{58}{2} = 29$

(c) No mode

(d) Range = Xm - X0 = 52 - 10 = 42

x^2	x
100	10
324	18
400	20
784	28
900	30
1296	36
1764	42
2704	52
$\Sigma x^2 = 8272$	$\Sigma x = 236$

$$S = \sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2} = \sqrt{\frac{8272}{8} - \left(\frac{236}{8}\right)^2}$$

$$= \sqrt{1034 - 870.25} = \sqrt{163.75} = 12.80$$

Q8.1: 20, 40, 60, 80, 100, 120, 140, 160, 180, 200.

Solution:

(a) Mean = $\frac{20 + 40 + 60 + 80 + 100 + 120 + 140 + 160 + 180 + 200}{10}$

$= \frac{1100}{10} = 110$

(b) Median = $\left(\frac{n+1}{2}\right)^{th}$ item = $\left(\frac{10+1}{2}\right)^{th}$ item = 5.5^{th} item

So, median = $\frac{100+120}{2} = 110$

(c) No mode

(d) Xm = 200, X0 = 20

Range = Xm= X0 = 200 - 20 = 180

(e) Standard deviation

x^2	x
400	20
1600	40
3600	60
6400	80
10000	100
14400	120
19600	140
25600	160
32400	180
40000	200
$\Sigma x^2 = 154000$	$\Sigma X = 110$

$$S = \sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2} = \sqrt{\frac{154000}{10} - \left(\frac{1100}{10}\right)^2}$$

$$= \sqrt{15400 - 12100} = \sqrt{3300} = 57.44$$

Q8.2: 5, 10, 40, 20, 35, 20, 50, 0, 5, 15, 25, 30, 20, 40, 45

Solution:

0, 5, 5, 10, 15, 20, 20, 20, 25, 30, 35, 40, 40, 45, 50

(a) Mean :

$$\frac{0 + 5 + 5 + 10\ 15 + 20 + 20 + 20 + 25 + 30 + 35 + 40 + 40 + 45 + 50}{15}$$

$$\text{Mean} = \frac{360}{15} = 24$$

(b) Median $= \left(\frac{n+1}{2}\right)^{th}$ item $= \left(\frac{15+1}{2}\right)^{th}$ item = 8thitem

So, Median = 20

(c) Mode = 20

(d) Range = Xm - X0 = 50 - 0 = 50

x^2	x
0	0
25	5
25	5
100	10
225	15
400	20
400	20
400	20
625	25
900	30
1225	35
1600	40
1600	40
2025	45
2025	50
$\Sigma x^2 = 154000$	$\Sigma x = 360$

$$S = \sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2} = \sqrt{\frac{12050}{15} - \left(\frac{\quad}{15}\right)^2}$$

$$\sqrt{\quad} \quad \sqrt{\quad}$$

Q8.3. Determine the mean, median, and mode for the following frequency distribution.

Solution:

C.F	C.B	FX	F	X
8	15-25	160	8	20
20	25-35	360	12	30
30 median class	35-45	400	$16 \rightarrow F_1$ $16 \rightarrow F_m$	40
46	45-55	800	$6 \rightarrow F_2$	50
52	55-85	480		80
-	-	$\Sigma FX = 2200$	$\Sigma F = 52$	-

$$\text{Mean} = \frac{\Sigma FX}{\Sigma F} = \frac{2200}{52} = 42.31$$

Median = $\ell = \frac{h}{f}\left(\frac{n}{2} - C\right)$, $\frac{n}{2} = \frac{52}{2} = 26$

$\ell = 35, h = 10, f = 10, \quad c = 20$

Median = $35 + \frac{10}{10}(26 - 20) = 35 + 1(6) = 35 + 6 = 41$

Mode = $\ell + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} + h$

$\ell = 45, F_m = 16, F_1 = 10, F_2 = 6, h = 10$

Mode = $45 + \frac{(16-10)}{(16-10)+(16-6)} \times 10 = 45 + \frac{6}{6+10} \times = 45 + \frac{6}{16} \times 10$

$= 45 + \frac{60}{16} = 45 + 3.75 = 48.75$

Q8.4: Forest Fire control table 14.18 summarizes data from an experiment in which the department of environmental management documented the number of forest fires reported each day over a 60 day period. Determine the mean, median, and mode for this data and interpret the meaning of each.

Solution:

FX	F	X
0	10	0
8	8	1
16	6	2
18	6	3
20	5	4
45	9	5
42	7	6
35	5	7
32	4	8
$\Sigma FX = 216$	$\Sigma x = 60$	–

Mean = $\frac{\Sigma FX}{\Sigma F} = \frac{216}{60} = 3.6$

Median = 4 (Middle value)

Mode = 0 (Value of X against highest frequency)

================

Q9:(A) Given a normal distribution where μ = 50 and μ = 8, determine the z values corresponding to each of the fallowing values of the random variable: (a) 56, (6) 42, (c) 66, (d) 36, and (e) 75.

Solution:

$\mu = 50, \quad \sigma = 8$

(a) $x = 56 \qquad z = \frac{x-\mu}{\sigma} = \frac{56-50}{8} = \frac{6}{8} = 0.75$

(b) $x = 42 \qquad z = \frac{x-\mu}{\sigma} = \frac{42-50}{8} = \frac{8}{8} = -1$

(c) $x = 66 \qquad z = \frac{x-\mu}{\sigma} = \frac{66-50}{8} = \frac{16}{8} = 2$

(d) $x = 36 \qquad z = \frac{x-\mu}{\sigma} = \frac{36-50}{8} = -\frac{14}{8} = -1.75$

(e) $x = 75 \qquad z = \frac{x-\mu}{\sigma} = \frac{75-50}{8} = \frac{25}{8} = 3.125$

(B) For the standard normal distribution determine:

(a) $P(z > 2.4)$ (b) $P(z < 1.2)$

(c) $P(0.8 < z < 3.0)$ (d) $P(-2.3 \le z \le 2.8)$

Solution:

$\mu = 0, \sigma = 1$

(a) $P(z > 2.4) = 0.5 - 0.4918 = 0.0082$

(b) $P(z < 1.2) = 0.5 + 0.3849 = 0.8849$

(c) $P(0.8 < z < 3.0) = P(0 < z < 3.0) - P(0 < z < 0.8)$

$= 0.49865 - 0.2881 = 0.2106$

(d) P (-2.3 z 2.8) = P (-2.3 z 0) + P (0 z 2.8)

$= 0.4893 + 0.4974 = 0.9867$

================

Q10: $= \lim_{x \to -9} \frac{81 - x^2}{9 + x}$

Solution:

$= \lim_{x \to -9} \frac{81 - x^2}{9 + x}$

$$= \lim_{x \to -9} \frac{(9)^2 - (x)^2}{(9+x)}$$

$$= \lim_{x \to -9} \frac{(9-x)-(9+x)}{(9+x)}$$

$$= \lim_{x \to -9} (9-x) = 9-(-9) = 9+9 = 18$$

Q10.1: $\lim_{x \to c} (4x^3 - 5x^2 + 10)$

Solution:

$$\lim_{x \to c} (4x^3 - 5x^2 + 10)$$

$$= 4c^3 - 5c^2 + 10$$

Q10.2: $\lim_{x \to -d} (x^2 - 2x + 3)$

Solution:

$$\lim_{x \to -d} (x^2 - 2x + 3)$$

$$= (-d)^2 - 2(-d) + 3 = d^2 + 2d + 3$$

For the following exercises, find the indicated limit and comment on the existence of any asymptotes.

Q10.3: $\lim_{x \to \infty} \frac{4}{x^2}$

Solution:

$$\lim_{x \to \infty} \frac{4}{x^2}$$

$$= \lim_{x \to \infty} \frac{4/x^2}{x^2/x^2} = \lim_{x \to \infty} \frac{4/x^2}{1}$$

$$= \frac{\lim_{x \to \infty} (4/x^2)}{\lim_{x \to \infty} (1)}$$

$$= \frac{4/\infty}{1} = \frac{0}{1} = 0 \qquad \text{(Say 'a')}$$

For horizontal asymptote:

$$\lim_{x\to\infty} f(x) = a$$

$$\Rightarrow \quad y = 0 \qquad \therefore a = 0$$

For vertical asymptote:

$$\lim_{x\to\infty} f(x) = \infty$$

$$\Rightarrow \frac{4}{x^2} = \infty$$

(Means numerator is not zero but denominator is zero).

$$\Rightarrow \quad x^2 = 0$$

$$x = 0$$

Q10.4: $\lim_{x\to\infty} \frac{5x-3}{x+10}$

Solution:

$$\lim_{x\to\infty} \frac{5x-3}{x+10}$$

$$= \lim_{x\to\infty} \frac{5\frac{x}{x} - \frac{3}{x}}{\frac{x}{x} + \frac{10}{x}}$$

$$= \lim_{x\to\infty} \frac{5 - \frac{3}{x}}{1 + \frac{10}{x}}$$

$$= \frac{5 - \frac{3}{\infty}}{1 + \frac{10}{\infty}} = \frac{5-0}{1+0} = \frac{5}{1} = 5 \text{ (Say a)}$$

For horizontal asymptote:

$$\lim_{x\to\infty} f(x) = a$$

$$\Rightarrow \quad y = 5 \qquad \therefore a = 5$$

For vertical asymptote:

$$\lim_{x\to\infty} f(x) = \infty$$

$$\Rightarrow \quad \frac{5x-3}{x+10} = \infty$$

$$\Rightarrow \quad x + 10 = 0$$

$x = -10$

Q10.5: $\lim\limits_{x\to\infty} \frac{-3x}{5x+100}$

Solution:

$$\lim_{x\to\infty} \frac{-3x}{5x+100}$$

$$= \lim_{x\to\infty} \frac{-3\,x/x}{5\,x/x + 100/x}$$

$$= \lim_{x\to\infty} \frac{-3x}{5x + 100/x} = \frac{-3}{5 + 100/x}$$

$$= \frac{-3}{5 + 100/\infty} = \frac{-3}{5+0} = -\frac{3}{5} \text{ (say a)}$$

For horizontal asymptote:

$$\lim_{x\to\infty} f(x) = a$$

$$\Rightarrow \quad y = -\frac{3}{5}$$

For vertical asymptote:

$$\lim_{x\to\infty} f(x) = \infty$$

$$\Rightarrow \quad \frac{-3x}{5x+100} = \infty$$

$$\Rightarrow \quad 5x + 100 = 0$$

$$5x = -100$$

$$x = -20$$

Q10.6: $\lim\limits_{x\to-\infty} \frac{8x+10}{-4x}$

Solution:

$$\lim_{x\to-\infty} \frac{8x+10}{-4x}$$

$$= \lim_{x\to-\infty} \frac{8\,x/x + 10/x}{-4\,x/x}$$

$$= \lim_{x \to -\infty} \frac{8 + 10/x}{-4} = \frac{8 + 10/\infty}{-4}$$

$$= \frac{8 + 10/\infty}{-4} = \frac{8}{-4} = -2 \qquad \text{(Say a)}$$

For horizontal asymptote:

$$\lim_{x \to -\infty} f(x) = a$$

$$\Rightarrow \quad y = -2$$

For vertical asymptote:

$$\lim_{x \to -\infty} f(x) = \infty$$

$$\frac{8x + 10}{-4x} = \infty$$

$$\Rightarrow \quad -4x = 0 = x = 0$$

================

Q11: $y = f(x) = -5x^2$

Solution:

$$y = f(x) = -5x^2$$

$$y_1 = f(-1) = -5(-1)^2 = -5$$

$$y_2 = f(2) = -5(2)^2 = -20$$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-20 - (-5)}{2 - (-1)} = \frac{-20 + 5}{2 + 1} = \frac{-15}{3} = -5$$

Q11.1: $y = f(x) = 3x^3 + 4x - 5$

Solution:

$$y = f(x) = 3x^3 + 4x - 5$$

$$y_1 = f(-1) = 3(-1)^3 + 4(-1) - 5$$

$$= -3 - 4 - 5 = -12$$

$$y_2 = f(2) = 3(2)^3 + 4(2) - 5$$

$$= 24 + 8 - 5 = 27$$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{27 - (-12)}{2 - (-1)} = \frac{27 + 12}{2 + 1} = \frac{39}{3} = 13$$

Q11.2: $y = f(x) = x^4$

Solution:

$$y = f(x) = x^4$$

$$y_1 = f(-1) = (-1)^4 = 1$$

$$y_2 = f(2) = (2)^4 = 16$$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{16 - 1}{2 - (-1)} = \frac{15}{3} = 5$$

Q11.3: $y = f(x) = x^4 - 10$

Solution:

$$y = f(x) = x^4 - 10$$

$$y_1 = f(-1) = (-1)^4 - 10 = 1 - 10 = -9$$

$$y_2 = f(2) = (2)^4 - 10 = 16 - 10 = 6$$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6 - (-9)}{2 - (-1)} = \frac{6 + 9}{2 + 1} = \frac{15}{3} = 5$$

================

Q12: $f(x) = ax + b$

Solution:

$$\frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \frac{\left[a(x+\Delta x)+b\right]-(ax+b)}{\Delta x}$$

$$= \frac{ax+a\Delta x+b-ax+b}{\Delta x}$$

$$= \frac{a\Delta x}{\Delta x} = a$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} a$$

$$\frac{dy}{dx} = a$$

Q12.1: $f(x) = ax^2 + bx$

Solution:

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$= \frac{\left[-a(x+\Delta x)^2+b(x+\Delta x)\right]-\left(-ax^2+bx\right)}{\Delta x}$$

$$= \frac{-a\left(x^2+2x\Delta x+(\Delta x)^2+bx+b\Delta x+ax^2-bx\right)}{\Delta x}$$

$$= \frac{-20x\Delta x-(\Delta x)^2+b\Delta x}{\Delta x}$$

$$= \frac{\Delta x(-20x-\Delta x+b)}{\Delta x} = -20x - \Delta x + b$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}(-20x-\Delta x+b)$$

$$\frac{dy}{dx} = -20x-0+b = -20x-b$$

Q12.2: $f(x) = -\frac{2}{x}$

Solution:

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$= \frac{\frac{-2}{x+\Delta x}-\left(-\frac{2}{x}\right)}{\Delta x}$$

$$= \frac{-\frac{2}{x+\Delta x}+\frac{2}{x}}{\Delta x}$$

$$= \frac{\frac{-2x+2(x+\Delta x)}{x(x+\Delta x)}}{\Delta x}$$

$$= \frac{-2x+2x+2\Delta x}{\Delta x.x(x+\Delta x)}$$

$$= \frac{2\Delta x}{\Delta x.x(x+\Delta x)}$$

$$= \frac{2}{x(x+\Delta x)}$$

$$\lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0}\frac{2}{x(x+\Delta x)}$$

$$\frac{dy}{dx} = \frac{2}{x(x+0)}$$

$$= \frac{2}{x.x} = \frac{2}{x^2}$$

Q13: $f(x) = \sqrt[3]{x^2-2x+5}$

Solution:

$$f(x) = \sqrt[3]{x^2-2x+5}$$

$$f(x) = (x^2-2x+5)^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}(x^2-2x+5)^{\frac{1}{3}-1}$$

$$\frac{d}{dx}(x^2-2x+5)$$

$$= \frac{1}{3}(x^2-2x+5)^{\frac{-2}{3}}\{2x-2(1)+0\}$$

$$= \frac{1}{3}(x^2-2x+5)^{\frac{-2}{3}}(2x-2)$$

$$= \frac{2x-2}{3(x^2-2x+5)^{\frac{2}{3}}}$$

Q13.1: $f(x) = \frac{1}{\sqrt{x^2-1}}$

Solution:

$$f(x) = \frac{1}{\sqrt{x^2-1}} = (x^2-1)^{-\frac{1}{2}}$$

$$f'(x) = -\frac{1}{2}(x^2-1)^{-\frac{1}{2}-1}\frac{d}{dx}(x^2-1)$$

$$= -\frac{1}{2}(x^2-1)^{-\frac{3}{2}}\{2x-0\}$$

$$= -\frac{1}{2}(x^2-1)^{-\frac{3}{2}}(2x)$$

$$= -\frac{x}{(x^1-1)^{\frac{3}{2}}} = -\frac{x}{\sqrt{(x^2-1)^3}}$$

Q13.2: $f(x) = \sqrt{\frac{1}{x^2+9}}$

Solution:

$$f(x) = \sqrt{\frac{1}{x^2+9}}$$

$$= \frac{1}{\sqrt{x^2+9}} = (x^2+9)^{-\frac{1}{2}}$$

$$f'(x) = -\frac{1}{2}(x^2+9)^{-\frac{1}{2}-1}\frac{d}{dx}(x^2+9)$$

$$= -\frac{1}{2}(x^2+9)^{-\frac{3}{2}}\{2x+0\}$$

$$= -\frac{1}{2}(x^2+9)^{-\frac{3}{2}}(2x) = -\frac{x}{(x^2+9)^{\frac{3}{2}}}$$

Q13.3: $f(x) = e^x$

Solution:

$$f(x) = e^x$$

$$f(x) = e^x\frac{d}{dx}(x) = e^x(1) = e^x$$

Q13.4: $f(x) = e^{x^2}$

Solution:

$$f(x) = e^{x^2}$$

$$f'(x) = e^{x^2}\frac{d}{dx}(x^2)e^{x^2}(2x) = 2xe^{x^2}$$

================

Q14: f (x) = (5x2 - 10)8

Solution:

$$f(x) = (5x^2 - 10)^8$$

$$f'(x) = 8(5x^2-10)^7\frac{d}{dx}(5x^2-10)$$

$$= 8(5x^2-10)^7\{5(2x)-0\}$$

$$= 8(5x^2-10)^7(10x) = 80x(5x^2-10)$$

$$= 80x(5x^2-10)^7$$

(a) **f' (x) =** $80(2)\left[5(2)^2-10\right]^7$

$$= 160(20-10)^7$$

$$= 160(20-10)^7 = (160)(10)^7$$

(b) **f' (x) = 0**

$$80x(5x^2-10) = 0$$

Either 80 x = 0 or

$$80x(5x^2-10) = 0$$

x = 0, 5x2 - 10 = 0 5x2 = 10

x2 = 2, x = $\pm\sqrt{2}$

Q14.1: f (x) = $(2x-8)^5$

Solution:

$$f(x) = (2x-8)^5$$

$$f'(x) = 5(2x-8)^4\frac{d}{dx}(2x-8)$$

$$= 5(2x-8)^4\{2(1)-0\}$$

$$= 5(2x-8)^4(2) = 10(2x-8)^4$$

(a) $f'(x) = 10[2(2)-8]^4$

$$= 10(4-8)^5 = 10(-4)^4$$

$$= (10)(256) = 2560$$

(b) $f'(x) = 0$

$$10(2x-8)^4 = 0$$

$$\Rightarrow \quad (2x-8)^4 = 0$$

$$\Rightarrow \quad 2x - 8 = 0,\ 2x = 8,\ x = 4$$

Q14.2: $f(x) = \sqrt{x^2+21}$

Solution:

$$f(x) = \sqrt{x^2+21}$$

$$f'(x) = \frac{1}{2}(x^2+21)^{-\frac{1}{2}}\{2x+0\}$$

$$= \frac{1}{2}(x^2+21)^{-\frac{1}{2}}(2x) = \frac{x}{\sqrt{x^2+21}}$$

(a) $f'(2) = \dfrac{2}{\sqrt{(2)^2+21}} = \dfrac{2}{\sqrt{4+21}}$

$$\frac{2}{\sqrt{25}} = \frac{2}{5}$$

Q14.3: $f'(x) = 0 \quad \dfrac{x}{\sqrt{2^2+21}} = 0$

Solution:

$$\Rightarrow \quad x = 0$$

Q14.4: $f(x) = e^{-x}$

Solution:

$$f(x) = e^{-x}$$

$$f'(x) = e^{-x}(-1)\ e^{-x}$$

(a) $f'(2) = -e^{-2} = \dfrac{1}{e^{-2}}$

(b) $f'(x) = 0$

$-e^{-x} = 0 \Rightarrow e^{-x} = 0$

Taking 'ln' on both sides.

$Ln(e^{-x}) = ln(0)$

$- \quad x \ln e = 0, -x(1) = 0$

$- \quad x = 0 \; x = 0 \quad \therefore \ln e = 1$

Q14.5: f (x) = - x2 + 4x + 15

Solution:

(a) **f (x) = - x2 + 4x + 15**

$f'(x) = -2x + 4(1) + 0$

$= -2x + 4$

$f'(x) = -2(1) + 4 = -2+4 = 2>0$

f is increasing function at x = 1

Now $f(x) = -x2 + 4x + 15$

$f'(x) = -2x + 4(1) + 0$

$= -2x + 4$

(b) **f will be an increasing function, when**

Solution:

$f'(x) > 0$

$-2x + 4 > 0$

$-2x > -4$

$2x < 4$

$x < 2$

(c) **f will be an increasing function, when**

Solution:

$f'(x) < 0$

$-2x + 4 < 0$

$-2x < -4$

$2x > 4$

$\mathbf{x > 2}$

(d) **f will be neither increasing function, mor decreaning, when Solution:**

$f'(x) = 0$

$-2x + 4 = 0$

$-2x = -4$

$2x = \frac{-4}{-2}$

$x = 2$

Q14.6:f (x) - 3x2 + 2x - 3

Solution:

f (x) = - 3x2 + 2x - 3

f' (x) = - 3 (2x) + 2 (1) - 0

= - 6x + 2

f" (x) = - 6 (1) + 0 = - 6

Put x = - 2 in f" (x), we get

f" (x) = - 6 < 0

The graph of f is concave down at

x = - 2. put x = 1 in f" (x),

we get

f" (x) = - 6 < 0

The graph of f is concave down at x = 1.

Q14.7:f (x) = x3 + 12x + 1

Solution:

f (x) = x3 + 12x + 1

f' (x) = 3x2 + 12 (1) + 0

= 3x2 + 12

f" (x) = 3 (2x) + 0 = 6x

Put x = - 2 in f" (x), we get

f" (-2) = 6 (-2) = - 12 < 0

The graph of f is concave down at x = - 2.

Put x = 1 in f" (x), we get

f" (1) = 6 (1) = 6 > 0

The graph of f is concave up at

x = 1.

================

Q15:(A) $f(x) = -2x^2 + \frac{x^4}{4}$

Solution:

$f(x) = -2x^2 + \frac{x^4}{4}$

$f'(x) = -2(2x) + \frac{4x^3}{4}$

$= -4x + 4x3$

$f''(x) = -4(1) + 4(3x2)$

$= -4 + 12x2$

Put $f'(x) = 0$

$= -4x + 4x3 = 0$

$= 4x3 - 4x = 0$

$= 4x(x2 - 1) = 0$

$= 4x(x-1)(x+1) = 0$

$= 4x = 0, x - 1 = 0, x + 1 = 0$

$= x = 0, x = 1, x = -1,$

Put x = 0 in f (x), we get

$f(0) = -2(0)2 + \frac{(0)^4}{4} = 0(0,0)$

Put x = 1 in f (x), we get

$f(1) = -2(1)^2 + \frac{(1)^4}{4}$

$= -2 + \frac{1}{4} = -\frac{7}{4} \notin \left(1, -\frac{7}{7}\right)$

The critical points are located at

$(0,0), \left(1, -\frac{7}{4}\right)\left(1, -\frac{7}{7}\right)$

Put (0,0) in f" (x), we get

$f(x) = -4 + 12(0) = -4 < 0$

The graph of f is concave down at x = 0 and a relative maximum at (0,0)

Put $\left(1, -\frac{7}{4}\right)$ in f" (x), we get

$f''(x) = -4 + 12(1) = -4 + 12 = 8 > 0$

The graph of f is concave up when

x = 1 and a relative minimum at $\left(1, -\frac{7}{4}\right)$

Put $\left(1, -\frac{7}{7}\right)$ in f" (x), we get

$f''(x) = -4 + 12(-1)$

$= -4 - 12$

$= -16 < 0$

The graph of f is concave down when x = - 1 and a relative minimum at

(B) A company estimates that the demand for its product fluctuates with the price it charges- The demand function is

`q = 280,000 - 400p

where q equals the number of units demanded and? equals the price in dollars. The total cost of producing q units of the product is estimated by he function

C = 350,000 + 300g + 0.0015q2

(a) Determine how many units 'q should be produced in order to maximize annual profit.

(b) What price should be charged?

(c) What is the annual profit expected to equal?

Solution:

$q = 280,000 - 400p$

$c = 350,00 + 300q + 0.0015q2$

Now

$q = 280,000 - 400p$

$400p = 280,000 - q$

$p = \frac{280,000}{400} - \frac{1}{400}q$

$p = 700 - 0.0025q$

We know that

$R = pq$

= (700 - 0.0025q)q

= 700q - 0.0025q2

Now

p = f (q)

p = R - C

p = (700q - 0.0025q2) - (350,000 + 300q + 0.0015q2)

p = 700 q - 0.0025q2 - 350,000 - 300q - 0.0015q2

f (q) p = - 0.00 40q2 + 400q - 350,000

(a)f (q) = - 0.0040 (2q) + 400 (1) - 0

= - 0.0080q + 400

Put f (q) = 0 - 0.0080q + 400 = 0

0.0080q = 400

$$q = \frac{400}{0.0080}$$

q = $ 50,000

(b) p = 700 - 0.0025 (50,000)

= $ 575

(c) p = - 0.0040 (50,000)2 + 400 (50,000) - 350,000

= $ 9,650,000